

Name KEY

## Chi-Square Practice

1. A poker-dealing machine is supposed to deal cards at random, as if from an infinite deck. In a test, you counted 1600 cards, and observed the following:

404 Spades      420 Hearts      400 Diamonds      376 Clubs

Is it equally likely to get a card from any of the 4 suits? Or are these discrepancies too much to be random?

- Write your null hypothesis
  - There is no significant difference between the expected and observed results
    - Any difference is due to chance
- Write your alternative hypothesis
  - There IS a significant difference between the expected and observed results
    - The differences are due to something other than chance
- Formalize your thinking by making a contingency box

	Spades	Hearts	Diamonds	Clubs
Exp	400	400	400	400
Obs	404	420	400	376

$$\frac{(404-400)^2}{400} + \frac{(420-400)^2}{400} + \frac{(400-400)^2}{400} + \frac{(376-400)^2}{400}$$

$$0.04 + 1 + 0 + 1.44$$

$$\text{Chi-square value} = 2.48$$

- Determine the critical chi-square value (use a p-value of 0.05)

p value	Degrees of Freedom							
	1	2	3	4	5	6	7	8
0.05	3.84	5.99	7.82	9.49	11.07	12.59	14.07	15.51
0.01	6.64	9.21	11.34	13.28	15.09	16.81	48.48	20.09

- Degrees of freedom =  $4 - 1 = 3$
- Critical chi-square value = 7.82
- Draw your conclusion
  - Our calculated chi-square value is less than our critical chi-square value
  - Therefore, we accept our null hypothesis which means
    - There is no statistically significant difference between observed and expected results
    - Any differences are probably due to chance alone

$$\chi^2 = \sum \frac{(\text{Observed Value} - \text{Expected Value})^2}{(\text{Expected Value})}$$

2. One study of grand juries in Alameda County, California, compared the demographic characteristics of jurors with the general population, to see if jury panels were representative. The results for age are shown below. The investigators wanted to know if the 66 jurors were selected at random from the population of Alameda County. (Only persons over 21 and over are considered; the county age distribution is known from Public Health Department data.) The study was published in the UCLA Law Review.

Age	County-wide %	# of jurors expected	# of jurors observed
21-40	42%	$.42 \times 66 = 27.72$	5
41-50	23%	$.23 \times 66 = 15.18$	9
51-60	16%	$.16 \times 66 = 10.56$	19
over 60	19%	$.19 \times 66 = 12.54$	33
Total	100%		66

Do we have evidence that grand juries are selected at random for this population?

- Write your null hypothesis
  - There is no significant difference between expected and observed results (For each age group, the proportion of jurors is consistent with the county proportion)
    - Any difference is due to chance
- Write your alternative hypothesis
  - There IS a significant difference between expected and observed results (For each age group, the proportion of jurors is NOT consistent with the county proportion)
    - Any difference is due to something other than chance

- Formalize your thinking by making a contingency box

- Calculate your chi-square value

Exp

Obs

	21-40	41-50	51-60	over 60
Exp	27.72	15.18	10.56	12.54
Obs	5	9	19	33

◦ Show your work:

$$\frac{(5-27.72)^2}{27.72} + \frac{(9-15.18)^2}{15.18} + \frac{(19-10.56)^2}{10.56} + \frac{(33-12.54)^2}{12.54}$$

$$18.62 + 2.52 + 6.75 + 33.38$$

◦ Chi-square value = **61.27**

- Determine the critical chi-square value (use a p-value of 0.05)

◦ Degrees of freedom =  $4 - 1 = 3$

◦ Critical chi-square value = **7.82**

- Draw your conclusion

◦ Our calculated chi-square value is greater than our critical chi-square value

◦ Therefore, we reject our null hypothesis which means

- There IS a significant difference between expected and observed results (the differences are due to something other than chance)
- We do NOT have evidence that grand juries in this population are selected at random (NOTE: Grand juries are nominated by judges, who tend to prefer older jurors)

3. In a study of the effectiveness of an antipsychotic drug, patients treated with the drug were compared to patients receiving a placebo. In terms of the number relapsing,

- 698 of 1,068 patients relapsed after taking the placebo
- 639 out of 2,127 patients relapsed after taking the antipsychotic drug

Test the prediction that the antipsychotic is significantly more effective in preventing relapses than the placebo.

- Write your null hypothesis
  - There is no significant difference between the effects of the antipsychotic drug and the placebo
    - Any difference is due to chance
- Write your alternative hypothesis
  - There IS a significant difference between the effects of the antipsychotic drug and the placebo
    - The antipsychotic drug is significantly more effective in preventing relapses than the placebo

- Formalize your thinking by making a contingency box

- Use antipsychotic drug numbers as observed results

- To calculate expected results:

- Determine the percentage of patients who did vs did not relapse in the control group (placebo)

$$698/1068 = 0.654 \text{ (\% relapsed in control group)}$$

$$(1068 - 698) / 1068 = 0.346 \text{ (\% did not relapse in control group)}$$

- Multiply the percentages by the total number of patients being treated with the antipsychotic drug

$$0.654 \times 2127 = 1391 \text{ (expected relapse)}$$

$$0.346 \times 2127 = 736 \text{ (expected no relapse)}$$

	Relapse	No Relapse
Exp	1391	736
Obs	639	1488 (2127-639)

- Calculate your chi-square value

$$\text{Show your work: } \frac{(639-1391)^2}{1391} + \frac{(1488-736)^2}{736} = 406.54 + 768.35$$

$$\text{Chi-square value} = 1174.89$$

- Determine the critical chi-square value (use a p-value of 0.01 this time)

$$\text{Degrees of freedom} = 2 - 1 = 1$$

$$\text{Critical chi-square value} = 6.64$$

- Draw your conclusion

- Our calculated chi-square value is greater than our critical chi-square value

- Therefore, we reject our null hypothesis which means

- There IS a significant difference between effects of the antipsychotic drug and the placebo (due to something other than chance)
  - The antipsychotic drug is significantly more effective in preventing relapses than the placebo